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Analysis of Nonlinear Mechanisms in a Precision Deployable Structure Using Measured Flexibility

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An experimental method of quantifying the nonlinear effects of varying preload on mechanisms within a deployable structure is presented. Local flexibilities are obtained from modal test data to examine the effects of the structure's nonlinear component mechanisms in situ. This technique allows the structural stiffness to be assessed separately from the structural mass so that variations in flexibility can be observed independent of the inertial loading of gravity off-load masses. Experimental application of the technique is also presented for a precision reconfigurable truss testbed. The variation of flexibility for three levels of off-load mass is studied to show the effect of gravity preload on mechanism stiffnesses. The results indicate flexibilities are more indicative of the effects of gravity preload than are individual modal parameters.

Nomenclature

$\{F\}$ = static force loading
$[G]$ = flexibility matrix
$[G_r]$ = residual flexibility matrix
$\{u\}$ = static displacement response

Subscripts

d = instrumented degrees of freedom (DOF) that are driving points (excitation and response)
n = measured modal DOF set
r = residual modal DOF set
s = instrumented DOF that are not driving points (response only)

Superscript

o = rank-deficient solution

Introduction

FUTURE deployable spacecraft structures must be designed to retain the structural rigidity and dimensional stability necessary for precision instrumentation. In contrast with most erectable spacecraft structures that have been investigated over the past 15 years, precision deployable structures are mechanically complex, nonlinear, time-variant dynamical systems. Design concerns for these structures will include deployment reliability, dimensional repeatability, and dynamic performance, all of which will require the development of high-fidelity models validated by ground test experiments. Examples of precision deployable structure concepts are included in Ref. 1.

The primary modeling challenge for precision deployable structures is the mechanical heterogeneity of nonlinear deployment mechanisms embedded within an otherwise linear structure. Wada et al.[2] reviewed a number of examples of the difficulties in ground-based testing of large precision space structures. Nonlinear variations of measured modal parameters with changes in excitation function and amplitude were shown to be a recurring problem. In response to this, Wada et al. explained how adaptive structures can relax ground test precision requirements.

Both ground and on-orbit testing of deployable, erectable, and rotary modules in the mid-deck 0-gravity dynamics experiment provided data on nonlinear variations in frequency, amplitude, and damping as functions of input amplitude, suspension stiffness, and preload.[3] Initial testing of the structure examined in this paper also focused on identifying nonlinear relationships between modal test input amplitudes and the subsequently identified modal parameters.[4]

This testing and analysis methodology provides direct measurement of the nonlinear effects of component mechanisms on global dynamic properties only once a structure is available for testing. In an attempt to provide more accurate foreknowledge of this sort of structure's dynamic character, a large amount of work in component mechanisms modeling as well as in the ensuing assembly process has taken place.

Belvin[5] examined the influence of joint stiffness and damping properties on global structural dynamics. A number of joint characterization techniques were tested and the component synthesis modeling approach was compared with that of finite elements. Additional research in nonlinear joint effects on space structure dynamics included examinations of modal coupling effects[6] and attempts to develop equivalent beam finite element models using an equivalent energy approach.[7] Bowden[8] examined the effects of a number of simple nonlinear joint models on global beam and truss dynamics.

Recently, a subset of the restoring force methods of nonlinear system identification termed force-state mapping (FSM) has been employed in joint characterization as well. Masters and Crawley[9] experimentally verified the FSM technique for multi-degree-of-freedom systems. With growing interest in the dimensional stability of optical precision deployables, this work has also been extended to observe micron-level joint mechanics.[10]

The work reported here lies somewhere between these joint modeling efforts and the global structural dynamics surveys mentioned earlier. A structural identification technique has recently been developed, which identifies the structural mass and stiffness characteristics separately and provides a measure of static flexibilities.[11,12] A fully reciprocal, but rank-deficient flexibility matrix for the structure

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is derived from measured modal frequencies, mode shapes, and residuals from modal tests using a limited set of excitation locations. Given the structural connectivity of the measured degrees of freedom (DOF), the flexibility matrix can then be disassembled to estimate the values of local stiffness parameters.[13]

This paper demonstrates how the use of this technique can provide more localized quantitative information than a purely modal approach. As a result, the in situ effects of deployment mechanisms can be observed more effectively and the accuracy of component models can be tested. Also, in separating the mass and stiffness characteristics of the structure, these tools allow inertial dead-mass off-loading of gravity forces using a simple pulley and weight suspension system.

To illustrate the flexibility approach, this paper describes an experiment performed on a batten actuated truss structure under a series of gravitational preload levels. The actuation mechanisms of this structure are preloaded by opposing springs to minimize backlash and free play in the structure. As a result, it is a precision adaptive truss capable of erectable structure precision. Prior experimentation on the test article has indicated a strongly nonlinear relationship between input amplitude and the measured modal response.[4] These nonlinear dynamic characteristics were expected to be sensitive to internal joint preload levels and have motivated further testing.

The objective of this experiment is to examine the effects of preload on the stiffness properties of deployment mechanisms using the identification techniques mentioned earlier. An inertial off-load system is used to present three structural preload cases. A series of modal surveys are performed for each case to acquire data for synthesizing the flexibility matrix. The experimental results indicate that measured flexibilities are highly sensitive to the effects of gravitational preloading on internal mechanisms. Therefore, it is suggested that such a ground test measurement can improve our knowledge of the contributions of individual mechanisms to deployable spacecraft stiffness.

Dynamically Measured Static Flexibility Matrix

The flexibility matrix $[G]$ relates the static deformation $\{u\}$ of a structure to a static force load vector $\{F\}$ according to

$$\{u\} = [G]\{F\} \tag{1}$$

For a restrained structure, the columns of $[G]$ represent the displacements of the structure under a static unit load applied at that column's DOF. These displacement shapes are also known as attachment modes.[14]

Prior measurements of the flexibility matrix addressed only the contribution of the identified vibration modes, referred to as the modal flexibility. However, the unidentified, or residual, modes also contribute a residual flexibility term. The residual flexibility represents the response of flexible modes that are not included in the identification's modal set. This includes modes that are above the test bandwidth or within the test bandwidth but poorly excited or unobservable by the given outputs. Generally, the residual flexibility is only on the order of 3–10% of the modal flexibility, but for applications requiring a high level of fidelity such as model refinement and damage detection, this factor can be very important.[15] In this test case, the residual flexibility is expected to be significant in resolving small changes in local flexibility as a result of variations in mechanism preloads.

Peterson and Alvin[11] have developed a method to estimate the residual flexibility in conjunction with the measured modes from measured frequency response functions (FRFs). This technique uses the pole information from a variant on the eigensystem realization algorithm to estimate the mode shapes concurrently with the residual flexibility and residual mass terms in the frequency domain.

Unfortunately, only a portion of the residual flexibility matrix is available directly from this technique. Only the partitions of the residual flexibility matrix $[G_r]$, relating the test excitation DOF $\{q_d\}$ and the test response DOF $\{q_s\}$, are available from the measured data. Measurements of the modal flexibility $[G_n]$, however, are defined between each pair of response DOF. This can be illustrated by partitioning $[G]$ according to the excitation and response DOF to obtain

$$[G] = \begin{bmatrix} G_{dd} & G_{ds} \\ G_{sd} & G_{ss} \end{bmatrix} \tag{2}$$

and then separating the modal and residual flexibilities as

$$[G] = \begin{bmatrix} G_{n_{dd}} & G_{n_{ds}} \\ G_{n_{sd}} & G_{n_{ss}} \end{bmatrix} + \begin{bmatrix} G_{r_{dd}} & G_{r_{ds}} \\ G_{r_{sd}} & G_{r_{ss}} \end{bmatrix} \tag{3}$$

As discussed earlier, all of $[G_n]$ and the left-hand partitions of $[G_r]$ are available from the identification. In addition, $[G_{r_{ds}}]$ is known to be the transpose of $[G_{r_{sd}}]$ because of reciprocity. Therefore, only the $[G_{r_{ss}}]$ partition of $[G_r]$ remains unidentified. However, it is important to note that $[G_{r_{ss}}]$ can be quite large whenever there are significantly more output than input locations, as is often the case.

A rank-deficient particular solution for $[G_{r_{ss}}]$ using only the measured data and preserving modal orthogonality is presented by Doebling et al.[16] Initially, a general solution for $[G_{r_{ss}}]$ is found to be

$$\left[G_{r_{ss}}\right] = \left[G_{r_{sd}}\right]\left[G_{r_{dd}}\right]^{-1}\left[G_{r_{sd}}\right]^T + [X][X]^T \tag{4}$$

where $[X]$ is an arbitrary matrix of dimension $(s \times r)$. This solution is then shown to satisfy modal orthogonality, and an argument is made to choose $[X] = 0$. This leads to a rank-deficient particular solution for $[G_{r_{ss}}]$ of the form

$$\left[G^o_{r_{ss}}\right] = \left[G_{r_{sd}}\right]\left[G_{r_{dd}}\right]^{-1}\left[G_{r_{sd}}\right]^T \tag{5}$$

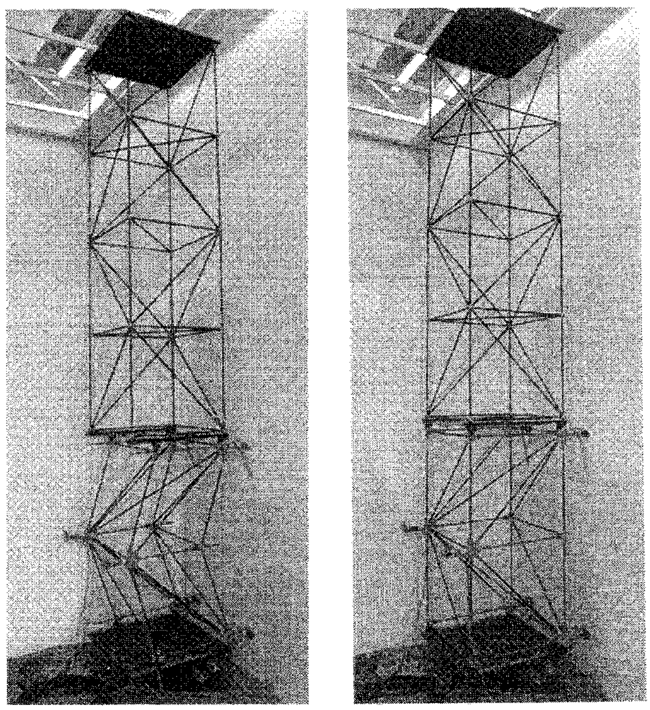
Finally, combining Eqs. (3) and (5) provides a rank-deficient estimate of the flexibility matrix,

$$[G^o] = [G_n] + \begin{bmatrix} G_{r_{dd}} & G^T_{r_{sd}} \\ G_{r_{sd}} & G^o_{r_{ss}} \end{bmatrix} \tag{6}$$

where $[G_n]$, $[G_{r_{dd}}]$, and $[G_{r_{sd}}]$ are identified from the measured FRF and $[G^o_{r_{ss}}]$ is computed using Eq. (5). Therefore, $[G^o]$ represents an estimate of the measured flexibility matrix including residual flexibility and requiring only the measured data.

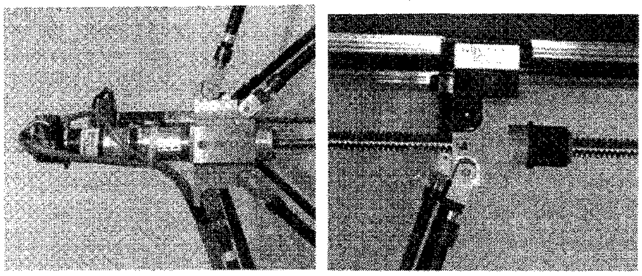
Test Article

The tested structure is shown in Fig. 1 in both partially and fully deployed configurations. The lowest two bays are precision batten



a) Partially deployed b) Fully deployed

Fig. 1 Test article: the batten actuated truss tower.



a) Pin-clevis hinges b) Linear bearings

Fig. 2 Batten actuated truss mechanisms.

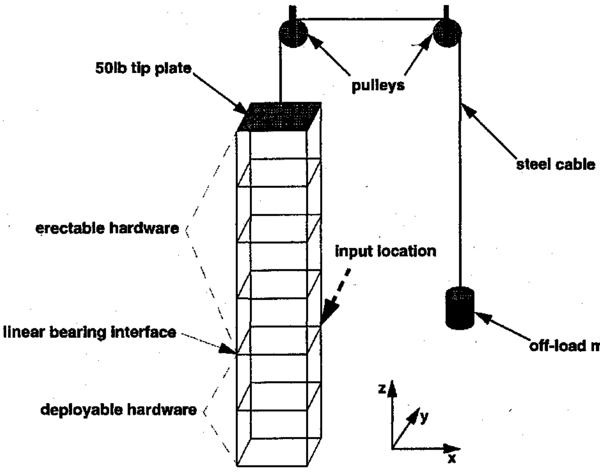


Fig. 3 Experiment configuration.

actuated truss hardware. Four bays of erectable truss hardware and a 50-lb tip plate are attached to increase both the gravitational preload and the global dynamics of the structure. Three pairs of screw jack actuators control the folding motion from a packaged to a fully deployed state (from 0.22 to 2.0 m end to end).

There are three types of mechanisms in this structure that are suspected sources of mechanical nonlinearity. Figure 2a shows a node with four pin-clevis hinges, which allow relative motion of the batten, diagonal, and longeron planes during deployment. These hinges consist of a press-fit steel pin and bronze bushing within the aluminum tang and clevis. The other mechanisms of interest are the linear bearings, which permit motion at the interfaces between the deployable structure and both the base and the erectable structure. In Fig. 2b, one of these mechanisms at the deployable/erectable interface is shown.

Finally, the actuation screws are secured to the structural nodes with opposing pairs of rotary bearings. These connections are contained within the housing shown in Fig. 2a and within the nut shown in Fig. 2b. With each of these mechanisms, increased loading is expected to lead to some increase in stiffness of the mechanisms' kinematic DOF.

Experimental Procedure

Inertial Off-Load Device

The inertial off-loading of the structure is provided by suspended masses supported by ceiling-mounted pulleys (see Fig. 3). A steel cable transmits the off-load to the 50-lb plate at the top of the structure. In examining the effects of preload, modal surveys of the structure were performed for three levels of off-loading: 0, 36.7, and 50 lb.

Modal Tests

The modal tests were performed on the fully deployed structure using a burst random input signal to obtain a structural response from 0 to 150 Hz. Triaxial acceleration measurements are made at each of the 32 nodes along with collocated force and acceleration measurements at the input location. Six additional pairs of horizontally oriented accelerometers are located on the motorized joints shown in Fig. 2a to capture a rocking behavior of the joints, which had been noted in prior testing.[4] These lead to a total of 104 acceleration output channels. Acquired spectra from 30 trials were averaged for each off-load case to estimate the FRFs.

Results

Sensitivity of Frequency Response Functions

The FRF data show relatively little variation as a result of the off-load level (see Fig. 4). Across the test bandwidth, only small variations in amplitude are apparent. Also, the differences that do exist reveal little or no consistent trend. From this, one would conclude that the FRF alone is not a reliable measure of the effects of mechanical nonlinearity.

Sensitivity of Identified Modal Parameters

The identified models consisted of 18 modes. Observing the first few identified modal frequencies in Table 1, there are both increases and decreases with off-load. In Table 2, increased identified damping with off-load is found for the first mode, but only slightly so for the second and third modes. Again, although the increased damping values may be found to be indicative of decreased joint preloading, there is little quantitative consistency in their variations.

Sensitivity of Synthesized Flexibility Matrices

We next examine the use of measured flexibilities in quantifying the effects of the mechanisms on the structure's mechanics. Figure 5 shows static flexibility shapes for loads at the input location and tip plate. Both of these shapes reveal the large amounts of bending-torsional coupling involved in the structure's static response.

Examining identified point flexibilities (the displacement at a sensor for a load at that sensor) in the structure reveals an increase in flexibility with increasing off-load. A global indicator of this trend is the average value of the point flexibilities for each case. These values, shown in Table 3, indicate a significant global increase in flexibility with increasing off-load.

Table 1 Modal frequencies

	Mode 1, Hz	Mode 2, Hz	Mode 3, Hz
0 off-load	18.64	34.67	38.09
36.7-lb off-load	18.48	34.47	38.49
50-lb off-load	18.45	34.40	38.44

Table 2 Modal damping ratios

	Mode 1, Hz, %	Mode 2, Hz, %	Mode 3, Hz, %
0 off-load	1.03	0.95	0.70
36.7-lb off-load	1.94	1.11	0.73
50-lb off-load	2.25	1.06	0.75

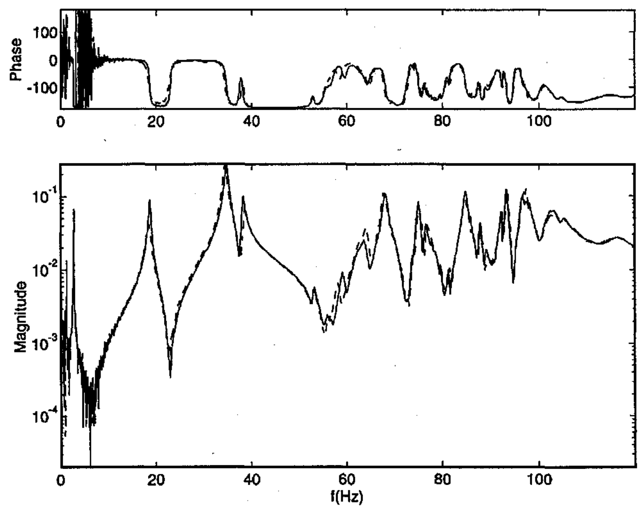


Fig. 4 Frequency response functions show little sensitivity to off-load: ——, 0 off-load and – – –, 50-lb off-load.

Table 3 Average point flexibilities

	Mean [diag (G)], m/N
0 off-load	1.51×10^{-6}
36.7-lb off-load	1.66×10^{-6}
50-lb off-load	1.82×10^{-6}

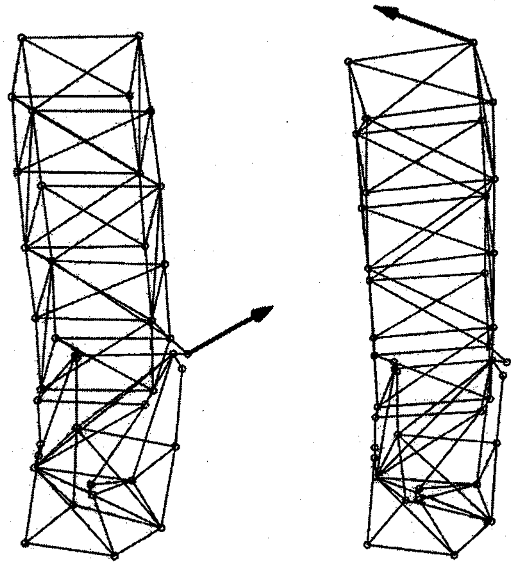


Fig. 5 Static flexibility shapes display strong bending-torsional coupling.

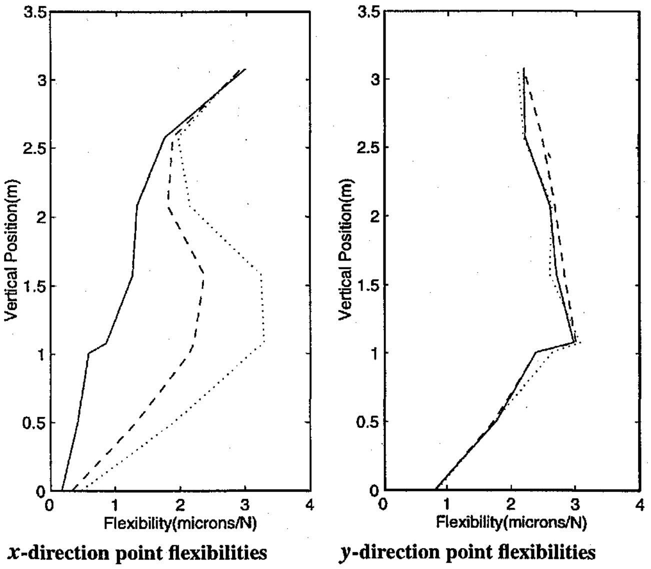


Fig. 6 Point flexibility trends along a vertical structural corner display sharp changes at linear bearing location (1 m): ——, 0 off-load; - - -, 36.7-lb off-load; and . . . ., 50-lb off-load.

Figure 6 portrays sets of point flexibilities along a vertical corner of the structure. A strong directional dependence in the off-load's effect is apparent. In the x direction (perpendicular to the input axis, see Fig. 3), this increase in flexibility is extreme, whereas there is little or no effect in the y direction. This directional dependency is consistent with the alignments of the deployment joints and the linear bearings at the structural interfaces. All deployment joints rotate about the x axis and may therefore be contributing directional preload dependencies to the structure's global flexibility.

The relatively large local flexibilities of the linear bearings are evidenced by a sharp increase in the point flexibilities in the y direction at the deployable/erectable interface location in Fig. 6. The interface

Table 4 Approximated local mechanism flexibilities for 0 off-load

Point flexibility between nodes	x direction, m/N	y direction, m/N
9 and 13 (linear bearing)	6.6×10^{-7}	7.7×10^{-7}
12 and 16 (rigid mounts)	2.7×10^{-7}	5.9×10^{-7}

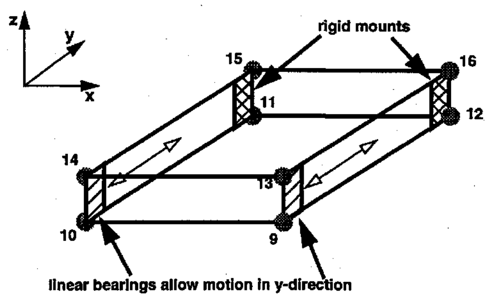


Fig. 7 Linear bearing locations at deployable/erectable structure interface.

is constrained only in the x direction by the linear bearing pair at nodes 9 and 10 as illustrated in Fig. 7. As a result, the decrease in stiffness across this mechanism is much greater in the y direction. Examining the increases in point flexibilities from nodes 9 and 12 to nodes 13 and 16, respectively, allows an approximate measure of the local stiffness contributions of the linear bearings and rigid mounts (see Table 4 and Fig. 7). As expected, the local increases in flexibility are slightly greater across the linear bearing location than across the rigid mount. Also, the increase in y-directional flexibility at the linear bearing is significantly greater than the increase in the x direction. Finally, the flexibility of the linear bearings is shown to be greater than that of the rigid mounts along the x direction.

Conclusions

Variations in both modal parameters and static flexibilities of a precision deployable structure were measured as a function of off-load. As seen in the modal parameter results from this experiment, it can be difficult to identify important effects of gravity off-load using identified modal data alone. In contrast, examining identified local flexibilities provides quantitative information about the performance of component mechanisms in situ. Initially, an increase in structural flexibility with increasing off-load was found to be dependent on direction. This directional dependence was consistent with the orientations of the structure's deployment and interface mechanisms. Also, sharp changes in flexibility along the structure indicated the high local flexibility of linear bearing mechanisms. Finally, estimates of these mechanisms' in situ flexibilities were made by measuring those local flexibility differences.

In summary, these results indicate that measured flexibilities are highly sensitive to the effects of gravitational preloading on internal mechanisms. Therefore, it is suggested that such a ground test measurement may lead to improved deployable spacecraft structural models by providing a more quantitative indication of the effect of deployment mechanisms on the structural mechanics.

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